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ABSTRACT

These materials are designed to be used as a tool to in-service school personnel who may wish to improve achievement levels on the mathematics objectives as measured by the Michigan Education Assessment Program (MEAP). Four areas of mathematics instruction (fractions, decimals, ratio and proportion, and percent) have been addressed in these materials. These areas were selected because of the low attainment rates exhibited on the 7th and 10th grade MEAP. The materials were prepared to assist teachers whose students are having difficulties in one or more of these areas. The materials include an analysis of the errors students make on MEAP, diagnostic tests, and teaching suggestions. (MP)

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MATHEMATICS

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FOREWORD

During the past year, in an effort to make state assessment data more relevant to the needs of educators throughout the state, a task force was formed to examine trends in state assessment over the past four years; and to investigate ways in which the Department could assist local school building personnel to use state assessment data in raising student achievement levels in reading and mathematics.

One of the major recommendations of the task force was that the School Program Services staff develop materials which could be used as a tool to in-service local school personnel in interpreting and using state assessment data to improve mathematics instruction.

The document, entitled "Michigan Education Assessment Program Support Materials for Mathematics" is now ready to be used to in-service staffs throughout the state who may wish to improve achievement levels on the mathematics objectives as measured by the Michigan Education Assessment Program.

John W. Porter
Superintendent of Public Instruction

Four areas of mathematics instruction, **Fractions, Decimals, Ratio and Proportion,** and **Percent,** have been addressed in these materials. These areas were selected because of the low attainment rates exhibited on 7th and 10th grade MEAP. The materials were prepared to assist teachers whose students are having difficulties in one or more of these areas.

The materials include:

1. an analysis of the errors students make on MEAP
2. diagnostic tests
3. teaching suggestions.

Any portion of the materials may be reproduced for instructional use.

CONTENTS

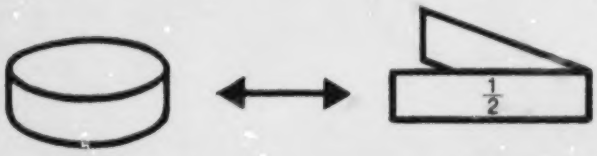
	pages
Fractions	
1. paper folding	1-a
2. concepts	2-a
3. addition	3-a
4. subtraction	4-a
5. multiplication	5-a
Decimals	
6. meaning	6-a
7. addition/subtraction	7-a
8. multiplication (metric)	8-a
9. multiplication (alternate)	9-a
10. division	10-a
Ratio and Proportion	11-a
Percent	12-a

FRACTIONS

PAPER FOLDING

The activities suggested within the fractions publications include folding of paper strips. Prior to using those materials it is necessary to be able to fold strips into equal parts.

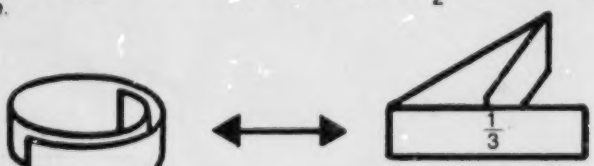
Two equal parts: Everyone can do this. Loop the strip once placing the ends together and crease.



Looped **Creased**

Label each part $\frac{1}{2}$

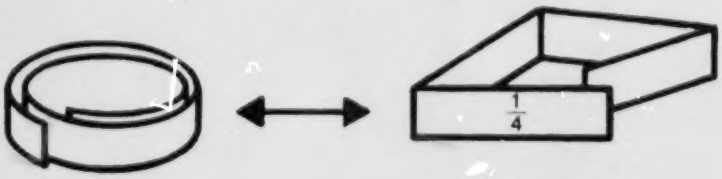
Three equal parts: Most people have done this. Loop the strip $1\frac{1}{2}$ times bringing the ends opposite one another and crease.



Looped **Creased**

Label each part $\frac{1}{3}$

Four equal parts: Most people do this by folding in half, creasing, and folding in half again. This is indeed $\frac{1}{2}$ of $\frac{1}{2}$ or $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. There is another technique which is an extension of the three-fold. Loop the strip twice, bringing the ends together and crease.

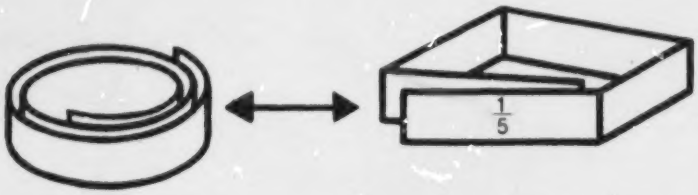


Looped **Creased**

How many equal parts? Four! Label each $\frac{1}{4}$. There are two hints for making this work well.

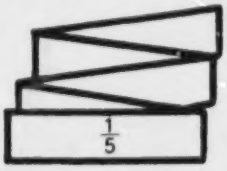
1. Keep the loops tight.
2. Squeeze together without creasing to be certain that the ends are at the same place.

Five equal parts: After folding into four equal parts using the method described above a five-fold should be easy. Loop the strip $2\frac{1}{2}$ times before creasing. Be certain that the ends of the strip are at the creases.



Looped **Creased**

How many equal parts? Five! Label each $\frac{1}{5}$. After folding the strip it is a good plan to "accordion fold" it. This gives the opportunity to check your accuracy and to mark the folds.



Accordion Fold

Six equal parts: Of course! There are three ways. Fold in half and then into thirds ($\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$). Or fold into thirds and then half ($\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$). Or use three loops.

Seven equal parts: Use $3\frac{1}{2}$ loops. Be sure the loops are tight!

More than seven equal parts: You're on your own.

Two sources are suggested for a supply of strips:

(1) a printer. Use strips that are left as scrap from trimming.

(2) legal sized pads can be cut length-wise to make strips that fold well.

FRACTIONS

CONCEPTS

RESULTS OF MEAP TESTING

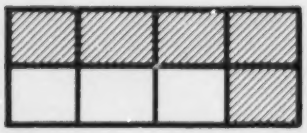
Four objectives are tested by the MEAP. Two objectives are tested at 7th grade and two at 10th grade.

7TH GRADE MEAP

Objective 1: Identify the shadded area — part to whole ratio.

sample test item

Choose the fraction that names the shaded part of the whole figure.



a. $\frac{3}{8}$

b. $\frac{5}{3}$

c. $\frac{5}{8}$

d. $\frac{3}{5}$

error analysis: On all but one item at least 70% of the students selected the correct response. The other 30% were fairly evenly split between choosing a part to part ratio and the incorrect part to whole ratio.

Objective 2: Order fractions with like denominators.

sample test item

Which group of fractions below is in order from smallest to largest?

(a) $\frac{5}{5}, \frac{4}{5}, \frac{3}{5}, \frac{2}{5}, \frac{1}{5}$

(b) $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}$

(c) $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{5}{10}, \frac{4}{10}$

(d) $\frac{5}{4}, \frac{4}{4}, \frac{2}{4}, \frac{2}{4}, \frac{1}{4}$

error analysis: Achievement on the items ranged from 66% to 75%. The two greatest distractors were descending order, as in (a), and ascending order for all but one fraction, as in (c).

10TH GRADE MEAP

Objective 1: Given a fraction, write a set of equivalent fractions.

sample test item

Which pair of fractions is equivalent to $\frac{5}{3}$?

(a) $\frac{25}{9}$, $\frac{10}{6}$

(b) $\frac{\cdot}{1}$, $\frac{5}{2}$

(c) $\frac{20}{12}$, $\frac{5}{4}$

(d) $\frac{15}{9}$, $\frac{30}{18}$

error analysis: Achievement ranged from 52% of the students selecting the correct response on one item to 68% on another. The best distractors always had one fraction which was equivalent to the fraction given. The square of the fraction is a good distractor.

Objective 2: Find the greatest of two fractions with unlike denominators.

sample test item

Mark the larger of the two fractions.

(a) $\frac{3}{4}$

(b) $\frac{5}{8}$

error analysis: Students did well on this objective. Item achievement ranged from 72% to 85%.

DIAGNOSTIC TEST

1. If a figure is divided into two parts the same size, the fraction name for each part is one-_____. The mathematical symbol is _____.

2. If a figure is divided into three parts the same size, the fraction name for each part is one-_____. The mathematical symbol is _____.

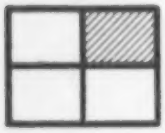
3. If a figure is divided into four parts the same size, the fraction name for each part is one-_____. The mathematical symbol is _____.

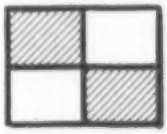
4. If a whole figure has *four* parts the same size and *three* of them are shaded, the mathematical symbol for the shaded portion is _____.

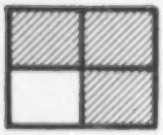
5. If a whole figure has *three* parts the same size and *two* of them are shaded, the mathematical symbol for the shaded portion is _____.

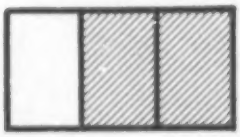
6. If a whole figure has *five* parts the same size and *three* of them are shaded, the mathematical symbol for the shaded portion is _____.

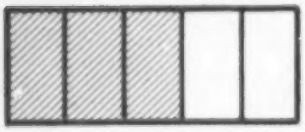
7. Write the fraction for the shaded part of each of these figures.

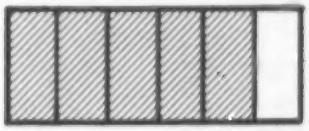
a. 

b. 

c. 

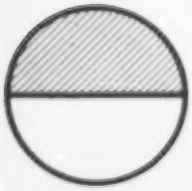
d. 

e. 

f. 

g.

h.

i. 

8. Draw a picture to show:

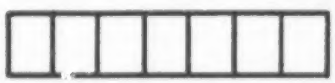
(a) $\frac{2}{3}$ (b) $\frac{4}{3}$

(c) $\frac{6}{3}$ (d) $\frac{8}{3}$

9. Draw a picture to show the mixed numbers:

(a) $1\frac{1}{4}$ (b) $2\frac{2}{5}$

10. Look at this figure:



(a) What is the largest fractional part that could be shaded?

(b) What is the smallest fractional part that could be shaded?

(c) Write the fractions that could be shown using the figure.

11. Place these fractions in order from smallest to largest: $\frac{10}{7}$, $\frac{3}{7}$, $\frac{5}{7}$, $\frac{4}{7}$, $\frac{7}{7}$.

12. Continue this sequence for the next three steps.

$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$

13. Continue this sequence by listing the next three equivalent fractions $\frac{2}{5}$, $\frac{4}{10}$, $\frac{6}{15}$, ____, ____, ____.

14. Write five equivalent fractions for $\frac{2}{3}$.

15. Change these fractions to twelfths.

(a) $\frac{1}{2} \cdot \frac{\square}{\square} = \frac{\square}{12}$ (b) $\frac{5}{3} \cdot \frac{\square}{\square} = \frac{\square}{12}$

(c) $\frac{2}{3} \cdot \frac{\square}{\square} = \frac{\square}{12}$ (d) $\frac{3}{4} \cdot \frac{\square}{\square} = \frac{\square}{12}$

(e) $3\frac{1}{2} = \frac{\square}{\square} \cdot \frac{\square}{\square} = \frac{\square}{12}$

(f) $1\frac{5}{6} = \frac{\square}{\square} \cdot \frac{\square}{\square} = \frac{\square}{12}$

TEST USE AND TEACHING SUGGESTIONS

Items 1-6. (Fractions — words and symbols.)

suggestion: Develop these concepts as paper folding techniques are developed for further work with fractions. Label each part with its fraction name. Stress that one of n equal parts has the fraction name 1/n, two have the name 2/n, etc.

Items 7-10. (Fractions — shaded ragions and symbols.)

suggestion: Continue the work suggested for items 1-6. However, students need to experience several models for fractional representations. Stress that there are n equal parts.

Items 11 and 16. (Ordering fractions — size comparisons.)

Difficulty with ordering when denominators are the same indicates a basic lack of understanding of the fraction concept.

suggestion: Fold and label paper strips stressing what the fraction means. ($\frac{2}{5}$ means two of five equal parts.)

Size comparisons are easily made using the strip folding model since direct comparisons are possible.

Items 12-15. (Equivalent fractions.)

suggestion: Equivalent fractions can be easily obtained by paper folding. In problem 12 each of the succeeding fractions are obtained by beginning with $\frac{1}{2}$ and folding each part into 2, 3, 4, 5, 6 equal parts. If $\frac{1}{2}$ is visualized as one of two equal parts and each part is folded into 6 equal parts, the corresponding portion will be 6 of 12 equal parts.

FRACTIONS

ADDITION

RESULTS OF MEAP TESTING

Three addition of fraction objectives were tested. Two objectives were tested at 7th grade and one at 10th grade.

7TH GRADE MEAP

Objective 1: Add fractions with like denominators.

sample test item

$$\begin{array}{r} \frac{5}{8} \\ +\ \frac{7}{8} \\ \hline \end{array}$$

error analysis: Approximately 20% of the students tested added both numerators and denominators.

Objective 2: Add mixed numbers with like denominators.

sample test item

$$\begin{array}{r} 4\ \frac{1}{3} \\ +2\ \frac{1}{3} \\ \hline \end{array}$$

error analysis: On the three items where regrouping was not necessarily nearly 75% of the students selected correct responses. Approximately 15% selected responses obtained by adding the whole numbers, adding numerators, and adding denominators. On the item involving fourths ($\frac{3}{4}$ and $\frac{1}{4}$) but requiring regrouping 61% selected the correct response. Selection of a response with denominators added was not possible. On the other item requiring regrouping 34% selected the correct response while 40% added the numerators and added denominators.

10TH GRADE MEAP

Objective 1: Add mixed numbers with like or unlike denominators.

sample test item

$$\begin{array}{r} 3\frac{1}{4} \\ +5\frac{3}{4} \\ \hline \end{array} \qquad \begin{array}{r} 2\frac{1}{2} \\ +1\frac{5}{6} \\ \hline \end{array}$$

error analysis: Four of the five test items did not contain a foil for the most common error — adding numerators and adding denominators. On those problems where regrouping was necessary there was significantly lower achievement.

DIAGNOSTIC TEST

1. $\frac{1}{4} + \frac{2}{4} =$ ____

2. $\begin{array}{r} \frac{3}{5} \\ +\frac{1}{5} \\ \hline \end{array}$

3. $\frac{3}{8} + \frac{4}{8} =$ ____

4. $\frac{7}{16} + \frac{5}{8} =$ ____

5. $\begin{array}{r} \frac{3}{8} \\ +\frac{2}{5} \\ \hline \end{array}$

6. $\begin{array}{r} \frac{1}{4} \\ +\frac{1}{6} \\ \hline \end{array}$

Rewrite in *vertical* form:

7. $\frac{4}{5} + \frac{2}{5} =$ ____

8. $\frac{2}{3} + \frac{3}{4} =$ ____

9. $1\frac{3}{4} + 2\frac{5}{6} =$ ____

Rewrite in *horizontal* form:

10. $\begin{array}{r} \frac{3}{7} \\ +\frac{2}{5} \\ \hline \end{array}$

11. $\begin{array}{r} 2\frac{5}{8} \\ +\frac{3}{5} \\ \hline \end{array}$

12. $\begin{array}{r} 4\frac{7}{8} \\ +3\frac{1}{3} \\ \hline \end{array}$

Write as a mixed number:

13. $\frac{8}{7} =$ ____

14. $\frac{9}{5} =$ ____

15. $\frac{19}{4} =$ ____

Addition of Fractions and Mixed Numbers:

16. $4\frac{1}{5} + 7\frac{2}{5} =$ ____

17. $\begin{array}{r} 3\frac{3}{5} \\ +2\frac{3}{5} \\ \hline \end{array}$

18. $\begin{array}{r} 6\frac{3}{8} \\ +5\frac{7}{8} \\ \hline \end{array}$

19. $\begin{array}{r} 10\frac{3}{8} \\ +4\frac{1}{3} \\ \hline \end{array}$

20. $3\frac{9}{16} + 2\frac{5}{6} =$ ____

21. $\begin{array}{r} 3\frac{4}{5} \\ +2\frac{3}{7} \\ \hline \end{array}$

TEST USE AND TEACHING SUGGESTIONS

Suggestion: Pick one model for representing fractions and use it exclusively. In these suggestions folding and labeling of paper strips is used.

Items 1-6. (Addition of fractions, like and unlike denominators.)

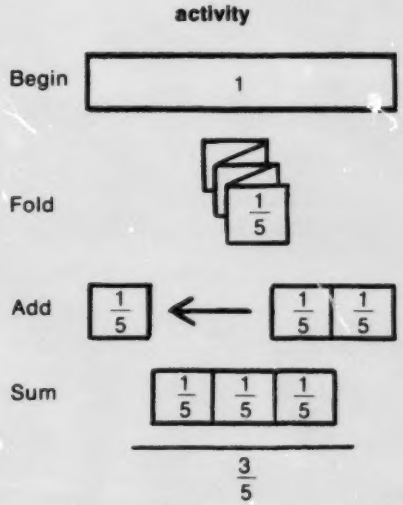
probably errors: Students add both numerators and denominators. This may be due to the lack of a mental image of the denominator.

suggestion
Fold, label and tape paper strips. Have the student work the problem using paper strips while writing the problem.

problem

$$\begin{array}{r} \frac{1}{5} \\ +\ \frac{2}{5} \\ \hline \end{array}$$

activity



Items 4-6. (Addition of fractions: unlike denominators.)

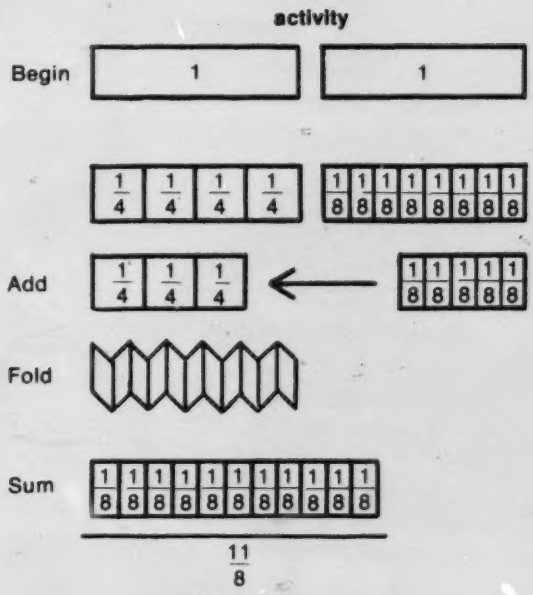
probable errors: Students will add both numerators and denominators.

suggestion
Fold, label, tape and fold paper strips.

problem

$$\begin{array}{r} \frac{3}{4} \\ +\ \frac{5}{8} \\ \hline \end{array}$$

activity



Items 7-12. (Changing problem format.)

probable errors: Many students compute only in vertical mode or only in horizontal mode. In transforming an exercise from one mode to another students are apt to copy incorrectly.

suggestion: Specific practice in rewriting exercises from one mode to the other.

Items 13-15. (Writing improper fractions as mixed numbers.)

This skill has pay-off when students add mixed numbers. Folding paper strips allows an easy comparison of an improper fraction and a mixed number in "simplest form."

Items 16-21. (Addition of mixed numbers.)

probable errors: Few students will have difficulty adding the whole numbers. However, all of the errors discussed previously with addition of fractions will occur with mixed numbers.

suggestion: Identify the specific problem and work with it. All of the paper folding activities suggested previously work with addition of mixed numbers.

FRACTIONS

SUBTRACTION

RESULTS OF MEAP TESTING

There are four subtraction of fractions objectives tested on the MEAP. Two of these objectives are tested at 7th grade and two at 10th grade.

7TH GRADE MEAP

Objective 1: Subtract a fraction from a mixed number with like denominators.

sample test item

$$3\frac{5}{6} - \frac{1}{6} = \underline{\quad}$$

error analysis: There is no consistent error pattern.

Objective 2: Subtract a fraction from a whole number.

sample test item

$$9 - \frac{5}{8} = \underline{\quad}$$

error analysis: On each of the test items approximately 20% of the children selected the response which was the whole number with the fraction appended (added) instead of subtracted.

10TH GRADE MEAP

Objective 1: Subtract fractions with unlike denominators.

sample test item

$$\frac{5}{6} - \frac{1}{3} = \underline{\quad}$$

error analysis: Approximately 25% of the students tested selected responses generated by either subtracting numerators and using the larger denominator ($\frac{4}{6}$) or subtracting numerators and subtracting denominators.

Objective 2: Subtract two mixed numbers with unlike denominators.

sample test item

$$2\frac{3}{8} - 1\frac{1}{2} = \underline{\quad}$$

error analysis: Approximately 40% of the students tested either subtracted the whole numbers; subtracted the denominators; and subtracted the numerators or subtracted the whole numbers and added the fractions.

DIAGNOSTIC TEST

1. $\begin{array}{r} \frac{3}{4} \\ -\frac{1}{4} \\ \hline \end{array}$

2. $\begin{array}{r} \frac{5}{6} \\ -\frac{3}{6} \\ \hline \end{array}$

3. $\begin{array}{r} \frac{7}{8} \\ -\frac{3}{8} \\ \hline \end{array}$

4. $\begin{array}{r} 3\frac{1}{2} \\ -\frac{1}{2} \\ \hline \end{array}$

5. $\begin{array}{r} 2\frac{2}{3} \\ -\frac{1}{3} \\ \hline \end{array}$

6. $\begin{array}{r} 4\frac{5}{8} \\ -\frac{3}{8} \\ \hline \end{array}$

7. How many thirds in 1?

8. How many fifths in 1?

9. How many sixths in 1?

10. $\begin{array}{r} 3 \\ -2\frac{1}{2} \\ \hline \end{array}$

11. $\begin{array}{r} 2 \\ -1\frac{5}{6} \\ \hline \end{array}$

12. $\begin{array}{r} 4 \\ -1\frac{3}{8} \\ \hline \end{array}$

13. $\begin{array}{r} \frac{1}{2} \\ -\frac{1}{3} \\ \hline \end{array}$

14. $\begin{array}{r} \frac{1}{2} \\ -\frac{1}{10} \\ \hline \end{array}$

15. $\begin{array}{r} \frac{5}{6} \\ -\frac{2}{3} \\ \hline \end{array}$

16. $\begin{array}{r} 3\frac{3}{4} \\ -\frac{1}{8} \\ \hline \end{array}$

17. $\begin{array}{r} 5\frac{5}{6} \\ -\frac{1}{2} \\ \hline \end{array}$

18. $\begin{array}{r} 2\frac{1}{4} \\ -\frac{7}{8} \\ \hline \end{array}$

19. $1\frac{1}{2} = \frac{\quad}{2}$

20. $1\frac{1}{3} = \frac{\quad}{3}$

21. $1\frac{4}{5} = \frac{\quad}{5}$

22. $\begin{array}{r} 3\frac{1}{2} \\ -1\frac{5}{6} \\ \hline \end{array}$

23. $\begin{array}{r} 3\frac{1}{2} \\ -2\frac{4}{5} \\ \hline \end{array}$

24. $\begin{array}{r} 2\frac{1}{4} \\ -1\frac{7}{8} \\ \hline \end{array}$

TEST USE AND TEACHING SUGGESTIONS

General Suggestion. Teach students to rewrite problems in vertical form.

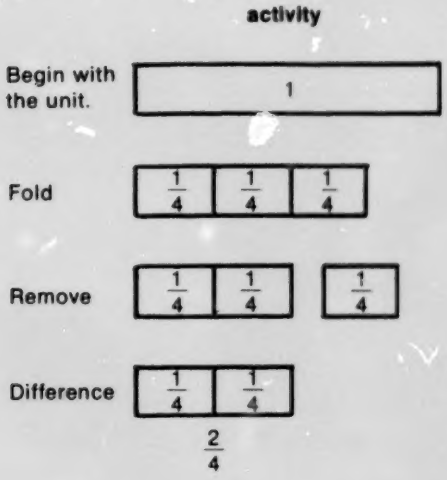
$\frac{5}{8} - \frac{3}{5} = \square$

$\begin{array}{r} \frac{5}{8} \\ -\frac{3}{5} \\ \hline \end{array}$

General Suggestion. Always inspect student work to determine the nature of the error.

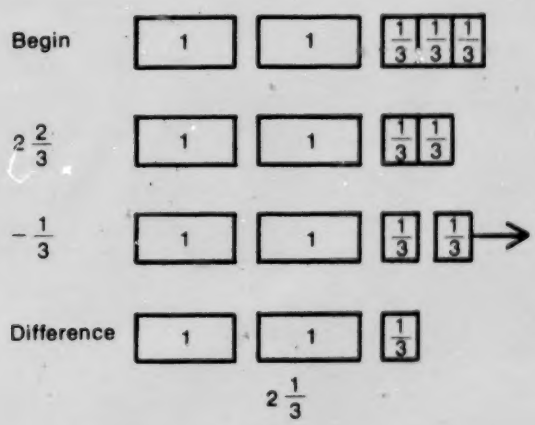
Items 1-3. (Subtract fractions with like denominators.)

probable errors: Students will probably either add the fractions or simply select the larger number.

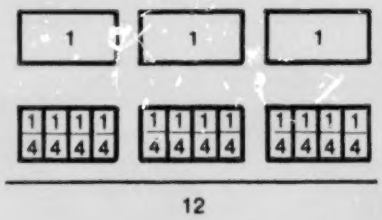
suggestion	**problem**	**activity**
Fold and label.	$\frac{3}{4}$ $-\frac{1}{4}$	

Items 4-6. (Subtract a fraction from a mixed number — like denominators.)

probable errors: 1. disregard of the fraction; 2. subtract numerators, subtract denominators; 3. student adds

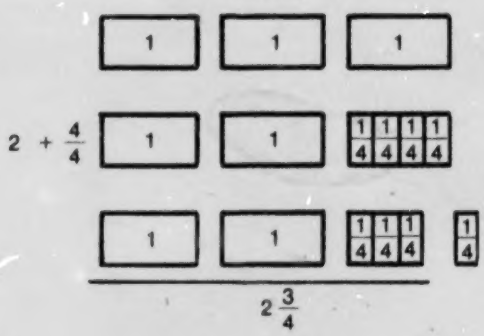
suggestion	**problem**	**activity**
Fold and label paper strips.	2 $-\frac{1}{3}$	

Items 7-9. (Changing whole numbers to fractional equivalents.)

suggestion	problem	activity
Fold paper strip. *Have the student label each part.*	How many $\frac{1}{4}$'s are in 3?	

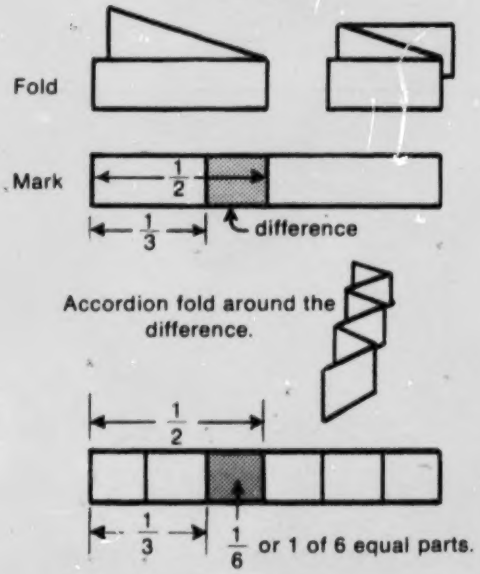
Items 10-12. (Subtract a mixed number from a whole number.)

probable errors: 1. regrouping is the major difficulty; 2. students will subtract the whole numbers and append the fraction; 3. students will subtract the numerator from the whole number.

suggestion	problem	activity
Fold and label paper strips.	$3 - \frac{1}{4}$	

Items 13-15. (Subtract fractions — unlike denominators.)

probable errors: By far the most common error made is subtracting numerators and subtracting denominators.

suggestion	problem	activity
Fold and label paper strips. The difficulty is with the "need for a common denominator".	$\frac{1}{2} - \frac{1}{3} =$ ____	

Items 16-18. (Subtract a fraction from a mixed number — unlike denominators.)

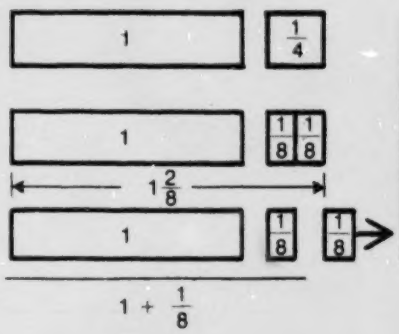
probable errors: Students will (1) add; (2) subtract numerators and denominators; (3) subtract numerators and use the largest denominator; (4) forget to regroup.

suggestion
Fold and label paper strips. Label each part.

problem

$$1\frac{1}{4} - \frac{1}{8}$$

activity



When regroup-ing is necessary this activity might be counter-productive for the standard algorithm but leads to a very good alternate algorithm.

Items 19-21. (Rewrite mixed numbers as improper fractions.)

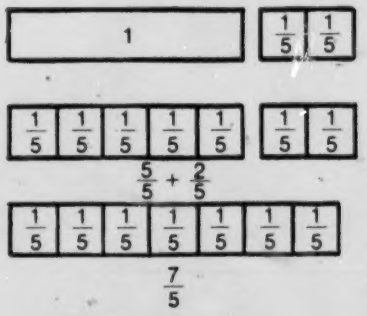
probable errors: The most common error is adding the whole number to the numerator ($4\frac{2}{5} = \frac{4+2}{5} = \frac{6}{5}$).

suggestion
Fold and label paper strips.

problem

$$1\frac{2}{5} \quad \frac{5}{5} + \frac{2}{5}$$

activity



Items 22-24. (Subtract mixed numbers — unlike denominators.)

probable errors: All of the difficulties encountered in the previous items will be found. However, regrouping substantially increases the difficulty. Many students will subtract correctly but forget to "borrow". ($3\frac{1}{3} - 2\frac{1}{2} = 1\frac{5}{6}$ instead of $3\frac{1}{3} - 2\frac{1}{2} = \frac{5}{6}$)

suggestion: Carefully analyze the students' work to determine the error being made and work on that mistake. After analyzing the students' work and determining the error, any of the previous activities could be used to correct the mistake.

FRACTIONS

MULTIPLICATION

RESULTS OF MEAP TESTING

The 7th grade assessment tests one objective on multiplication of fractions and the 10th grade assessment tests three objectives.

7TH GRADE MEAP

Objective: Multiply two unit fractions.

sample test item

$$\begin{array}{r} \frac{1}{3} \\ \times \frac{1}{5} \\ \hline \end{array}$$

error analysis: On the five items measuring this objective between 20% and 29% of the students selected responses obtained either by adding numerators and multiplying denominators or by multiplying numerators and adding denominators.

10TH GRADE MEAP

Objective 1: Multiply two fractions with denominators less than 7.

sample text item

$$\frac{2}{3} \times \frac{4}{5} = ____$$

error analysis: Correct responses for the four items ranged from 85% to 93%. Approximate 10% of the students selected responses in which either numerators or denominators were added. Products were not reduced.

Objective 2: Multiply a whole number less than 5 and a fraction with a denominator less than 7.

sample test item

$$4 \times \frac{2}{3} = ____$$

error analysis: Achievement was over 70% on each of the items. Between 7% and 8% multiplied the whole number and the denominator. Between 14% and 16% of the students thought that both the numerator and denominator should be multiplied by the whole number.

Objective 3: Multiply a whole number and a mixed number.

sample text item

$$4 \times 3\frac{2}{3} = ____$$

error analysis: Achievement on these items was very low: 44%-46%. Nearly as many students (39%-44%) multiplied only the whole numbers and appended the fraction. Regrouping was necessary on each item.

DIAGNOSTIC TEST

1. $\frac{1}{3}$ of 6 = ____
2. $\frac{1}{2}$ of 9 = ____
3. $\frac{2}{3}$ of 7 = ____
4. $\frac{1}{3}$ of $\frac{2}{5}$ = ____
5. $\frac{3}{8}$ of $\frac{4}{2}$ = ____
6. $\frac{8}{9}$ of $\frac{3}{2}$ = ____
7. $\frac{1}{2} \times \frac{1}{5}$ = ____
8. $\frac{2}{3} \times \frac{1}{2}$ = ____
9. $\frac{3}{4} \times \frac{3}{5}$ = ____
10. $\frac{5}{3} \times \frac{5}{2}$ = ____
11. $\frac{3}{2} \times \frac{5}{3}$ = ____
12. $\frac{10}{7} \times \frac{14}{5}$ = ____
13. $4 \times 3\frac{2}{3}$ = ____
14. $\frac{3}{4} \times 5$ = ____
15. $1\frac{2}{3} \times 4$ = ____
16. $2\frac{2}{3} \times 1\frac{1}{4}$ = ____
17. $2\frac{3}{5} \times 3\frac{5}{6}$ = ____
18. $4\frac{3}{8} \times 3\frac{1}{2}$ = ____
19. When multiplying by $\frac{17}{19}$ will the number be larger or smaller?
20. When multiplying by $\frac{21}{19}$ will the number be larger or smaller?

TEST USE AND TEACHING SUGGESTIONS

Remark I: Using the "of" interpretation for multiplication of fractions can be illustrated nicely by paper folding. Paper folding, or another concrete model, will provide the student a conceptual model which will assist in answering problems 19 and 20 above.

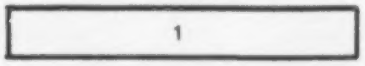
Remark II: Multiplication of fractions and reduction of fractions to "lowest terms" are two *separate* skills. If the skill of multiplication is being tested, the students should not be required to reduce to lowest terms. Once students have demonstrated their ability to multiply fractions it is appropriate to check their ability to reduce to lowest terms.

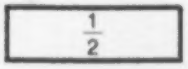
Reduction of fractions to "lowest terms" is not always desirable. It should never be required of students as a reflex action. Multiplication of fractions exercises should be scored as correct if they are correctly given in either "higher terms" or "lowest terms".

Items 1-6. (A fraction "of" a whole number or fraction.)

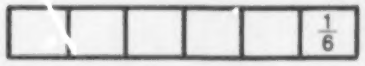
These problems have been included because the "of" model is the basis of the paper folding technique suggested in this publication.

suggestion	problem	activity
Paper fold.	$\frac{1}{3}$ of $\frac{1}{2}$	

Begin 

Fold and label $\frac{1}{2}$ (Fold the other $\frac{1}{2}$ behind) 

Fold into thirds $\frac{1}{3}$ of $\frac{1}{2}$



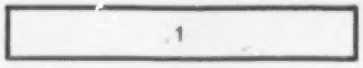
Unfold and determine the part of the strip you have.

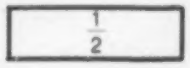
Items 7-12. (Fraction multiplied by a fraction.)

probable errors: Students will feel compelled to add somewhere. Either they will add the numerators and multiply the denominators or they will multiply the numerators and add the denominators.

suggestion: Provide opportunities to multiply by fractions both larger and smaller than one. Encourage them to make generalizations.

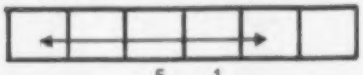
suggestion	problem	activity
Fold and label paper.	$\frac{5}{3} \times \frac{1}{2}$	

Begin 

$\frac{1}{2}$ 

Fold $\frac{1}{3}$ of $\frac{1}{2}$

Unfold and count.



$\frac{5}{3} \times \frac{1}{2}$

Compare with $\frac{1}{2}$. Is $\frac{5}{3} \times \frac{1}{2}$ larger or smaller than $\frac{1}{2}$? Is $\frac{5}{3} \times \frac{1}{2}$ larger or smaller than one?

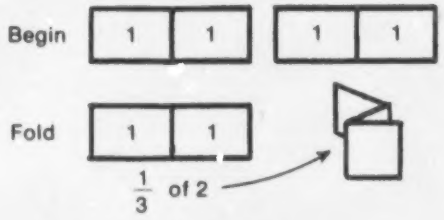
Items 13-15. (Whole number multiplied by a fraction or a mixed number.)

probable errors: The most common error is to multiply both numerator and denominator by the whole number. With mixed numbers another common error is to multiply the whole numbers and append the fraction.

suggestion
Fold and tape paper strips.

problem
$1\frac{1}{3} \times 2$

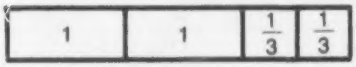
activity



Think 1 of 2 & $\frac{1}{3}$ of 2.

Refolding is necessary to name $\frac{1}{3}$ of 2.

$1\frac{1}{3}$ of 2



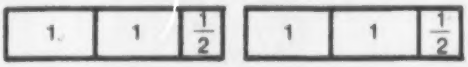
Items 16-18. (Multiplication of mixed numbers.)

probable errors: All of the errors made with multiplication of fractions will appear in these problems.

suggestion
Paper folding for multiplication of mixed numbers works very well and reinforces all of the skills of multiplication of fractions.

problem
$1\frac{1}{3} \times 2\frac{1}{2}$

activity



Think $\frac{1}{3}$ of $2\frac{1}{2}$

Refolding is necessary for renaming — compare with the portion of the unit folded behind.

$2\frac{1}{2}$ $\frac{1}{3}$ of $2\frac{1}{2}$

Items 19-20. (Multiplication by fractions larger and smaller than one.)

The student needs many experiences with multiplying by fractions both larger and smaller than one and needs to be encouraged to make generalizations such as:

In multiplying two numbers, if the first number is:

(a) greater than 1, the product will be greater than the second number.
(b) equal to 1, the product will be the same as the second number.
(c) less than 1, the product will be smaller than the second number.

DECIMALS

MEANING

RESULTS OF MEAP TESTING

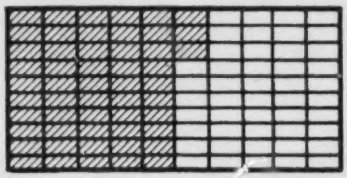
Five objectives are tested on the MEAP. Two of the objectives are tested at 7th grade and three are tested at 10th grade.

7TH GRADE MEAP

Objective 1: Identify illustrated decimal fraction.

sample test item

Choose the decimal that names the shaded part of each whole figure.



(a) .29
(b) .37
(c) .47
(d) .53

error analysis: Students did very well on this objective. Item scores ranged from 77%-86% of the students selecting the correct response.

Objective 2: Name place values of decimal fractions.

sample test item

In .923, the 3 means

(a) 3 thousand
(b) 3 thousandths
(c) 3 hundred
(d) 3 hundredths

error analysis: Item achievement ranged from 39%-43%. This is significantly above objective achievement which indicates a great deal of guessing. It appears that many students determine decimal place value by starting from the right — neglect the decimal points and add a "ths" to the whole number place value.

10TH GRADE MEAP

Objective 1: Round to the nearest whole number, tenths, or hundredths.

sample test item

100.89 rounded to the nearest tenth is:

(a) 100
(b) 101
(c) 100.8
(d) 100.9

error analysis: Achievement is above 80% on those items requesting rounding to the nearest whole number, 50% on those items requesting rounding to tenths and hundredths. On the latter items students continue to round to the nearest whole number.

Objective 2: Rename common fractions as decimal fractions.

sample test item

What decimal is equivalent to $\frac{2}{5}$?

(a) .40
(b) .25
(c) .20
(d) 2.5

error analysis: Item achievement ranged from 37%-61%. The most commonly selected foils were obtained by using the two digits from the fraction with a decimal point inserted somewhere.

Objective 3: Arrange fractions in ascending or descending order.

sample test item

Which set of fractions is in order from least to greatest?

(a) 0.66, 0.64, 0.65, 0.63
(b) 0.65, 0.66, 0.64, 0.63
(c) 0.63, 0.64, 0.65, 0.66
(d) 0.63, 0.66, 0.64, 0.65

error analysis: The errors increased directly with the increase in the numbers of digits which changed, and with the introduction of nonconsecutive numbers.

DIAGNOSTIC TEST

For items 1-4, choose the decimal that names the shaded part of the figure.

1.

1 meter

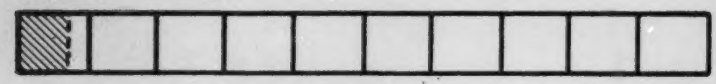
(a) 3.0
(b) 0.3
(c) 0.7
(d) 7

2.

1 meter

(a) .25
(b) .2
(c) 7.5
(d) 2.5

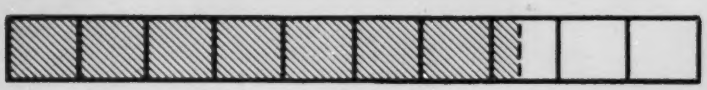
3.



1 meter

(a) .7
(b) 9.3
(c) .93
(d) .07

4.



1 meter

(a) .74
(b) .26
(c) .7
(d) 2.6

for items 5-8, choose the correct *word* answer.

5. In .2, the 2 means:
   (a) 2 ones
   (b) 2 tens
   (c) 2 tenths
   (d) 2 twos

6. In .04, the 4 means:
   (a) 4 tenths
   (b) 4 hundredths
   (c) 4 hundreds
   (d) 4 ones

7. In .673, the 3 means:
   (a) 3 hundreds
   (b) 3 tenths
   (c) 3 thousandths
   (d) 3 ones

8. In 3.527, the 2 means:
   (a) 2 tens
   (b) 2 thousandths
   (c) 2 hundredths
   (d) 2 thousands

9. 3.2 rounded to the nearest whole number is:
   (a) 3
   (b) 3.2
   (c) 4
   (d) 3.3

10. 64.8 rounded to the nearest whole number is:
    (a) 64
    (b) 64.9
    (c) 65
    (d) 64.8

11. 3.79 rounded to the nearest tenth is:
    (a) 3
    (b) 3.8
    (c) 3.70
    (d) 4

12. 2.474 rounded to the nearest hundredth is:
    (a) 2.47
    (b) 2.474
    (c) 2.5
    (d) 2

13. Which decimal is equal to $\frac{1}{2}$?
    (a) 1.2
    (b) 0.12
    (c) 0.5
    (d) 5

14. Which decimal is equal to $\frac{3}{4}$?
    (a) 0.75
    (b) 3.4
    (c) 0.34
    (d) 7.5

15. Which decimal is equal to $\frac{4}{1000}$?
    (a) 0.0004
    (b) 0.004
    (c) 0.04
    (d) 0.4

16. What is another name for $\frac{3}{5}$?
    (a) 0.60
    (b) 0.35
    (c) 3.5
    (d) .15

17. Which set of decimal fractions is in order from *least* to *greatest*?
    (a) 0.35 0.34 0.36 0.37
    (b) 0.34 0.35 0.36 0.37
    (c) 0.37 0.36 0.35 0.34
    (d) 0.36 0.37 0.34 0.35

18. Which set of decimal fractions is in order from *least* to *greatest*?
    (a) 37.8 37.9 38.0 38.1
    (b) 38.1 38.0 37.9 37.8
    (c) 37.9 38.0 37.8 38.1
    (d) 38.1 37.8 37.9 38.0

19. Which set of decimal fractions is in order from *least* to *greatest*?

(a) 3.01 2.11 2.10 2.01
(b) 2.11 3.01 2.01 2.10
(c) 2.01 3.01 2.10 2.11
(d) 2.01 2.10 2.11 3.01

20. Which set of decimal fractions is in order from *least* to *greatest*?

(a) 0.47 0.8 0.17 0.9
(b) 0.17 0.8 0.9 0.47
(c) 0.17 0.47 0.8 0.9
(d) 0.8 0.9 0.17 0.47

TEST USE AND TEACHING SUGGESTIONS

Suggestion 1. Approach the meaning of decimals through actual experiences in measurement with a meter tape or a centimeter ruler.

Suggestion 2. Develop understanding of decimal numeration through the linear model suggested above and constant reference to the "grouping of tens" principle which continues across the decimal point. Emphasize that place value relationships in decimal numeration are the same as in whole number numeration: each place value is 10 times the value of the place to its right, and $\frac{1}{10}$ the value of the place to its left. (Only the *names* appear to reverse.)

Suggestion 3. Develop the "rounding" concept from visual observation of the meter model and from the experience of *approximate* measurement.

Suggestion 4. Develop the concept of renaming initially through decimal equivalents (e.g. .5 = .50 = .500) using comparative meter-number lines.

Suggestion 5. Extend the concept of renaming common fractions to decimal fractions. Using paper folding techniques and paper strips of meter and decimeter length, fold in appropriate parts, mark, fold, observe metric measurement, record decimal numeration.

Suggestion 6.

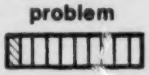
(a) Develop the concept of order in decimals through *counting* with various metric units and subunits, writing, reading, and comparing these units and subunits on appropriately marked meter models.

(b) Repeated comparisons to counting, grouping, and order in the whole number system should facilitate this extension of the order concept.

(c) Exercises of the "more than" and "less than" type with decimal fractions should precede requiring the order sequence of more than two decimal numbers.

Items 1-4. (Identify illustrated decimal fraction.)

probable errors: Students will name the *unshaded* part.

suggestion	problem	activity
Emphasize the *shaded* part by measuring and shading in actual measurements. Become familiar with a linear model through use of meter, decimeter, etc.	 (a) 1.0 (b) 0.1 (c) 0.9 (d) 9.0	Measure objects with marked meter strips of paper. Mark and shade in the actual length of objects. Label answers with the correct decimal name.

Items 5-8. (Name the place value of decimal fractions.)

probable errors: Students will select either the whole number place value without regard to the decimal or the correct place value without the "ths".

suggestion	problem	activity
Use the meter model which demonstrates dramatically the reality of tenths, hundredths and thousandths as successively *smaller* parts of one whole unit.	In .673 the 3 means: 3 hundreds 3 tenths 3 thousandths 3 ones	• Measure many objects in meters, then decimeters, centimeters and millimeters. • The move from approximate measurement to increasingly more accurate measurement demonstrates the comparative value of these subunits. • Recording the results necessitates decimal numeration. Recording in different ways teaches equivalence: (2 dm + 3 cm) or (23 cm).

Items 9-12. (Round to the nearest whole number, tenths or hundredths.)

probable errors: Students will most likely choose the nearest *whole number* instead of the nearest tenths or hundredths as requested. Students may also select a whole number which is *not* the nearest.

suggestion	problem	activity
Use the metric linear model with its various subunits to visually recognize which is the nearest tenth, hundredth, or whole number.	Round to nearest whole number: 1.3	• Begin with a 2 meter model. • Mark location of 1 m, 2 m. • Mark location of 1.3 m. • Observe and record whether it is closer to 1 m or 2 m.
	Round to nearest tenths: .27	• Begin with a meter model marked in cm and dm. • Mark location of 2 dm and 3 dm. • Locate .27 m and observe whether it is closer to 2 dm or 3 dm.
	Round to nearest hundredths: .341	• Begin with a meter model marked in dm, cm, mm. • Mark location of 34 cm and 35 cm. • Locate .341 m and observe whether it is closer to .34 m or .35 m.

Items 13-16. (Rename common fraction as decimal fraction.)

probable errors: Students who do not recognize that the given common fraction is less than one may choose 1.2, while those who do recognize that concept but don't understand decimal place value, will choose .12.

suggestion	problem	activity
Use paper folding and comparative linear models. Limit experiences to halfs, fifths, and tenths initially. Then develop with fourths and thirds.	$\frac{1}{2} = ?$ (a) 1.2 (b) 0.12 (c) 0.5 (d) 5	Fold a paper strip marked in decimeters in half. Unfold and count the decimeters in each part. Record: $\frac{1}{2}$ of the meter = 5 dm = 5 tenths of a meter = 0.5 m.
	$\frac{1}{4} = ?$	Fold a paper strip marked in decimeters and centimeters into fourths. Unfold and count the decimeters and centimeters. Record: $\frac{1}{4}$ of the meter = 2 dm and 5 cm = 25 cm = 25 hundredths of a meter = .25 m.

Items 17-20. (Arrange decimal fractions in order from *least* to *greatest*.)

probable errors: Students will be confused by nonconsecutive numbers and by changes in more than one place value position.

suggestion	problem	activity
Use the metric model to locate, compare and count.	Teaching sequence: 1. Within tenths .3, .4, .5, .6 2. Whole number and tenths 2.6, 2.7, 2.8, 2.9, 1.8, 1.9, 2.0, 2.1 3. Tenths and hundredths .22, .23, .24, .25, .47, .48, .49, .50 4. Whole number and hundredths 1.06, 1.07, 1.08, 1.09, 6.08, 6.09, 6.10, 6.11 5. Whole number and hundredths 7.98, 7.99, 8.00, 8.01	Teach students to compare by whole number first, then by hundredths. Order by place value *first*, then *within* place value.

DECIMALS

ADDITION/SUBTRACTION

RESULTS OF MEAP TESTING

There are three objectives tested — all of them on the 7th grade test.

7TH GRADE MEAP

Objective 1: Addition or subtraction — tenths.

sample test item

Emma walks to and from school. It is .4 kilometers from Emma's house to school. How far does Emma walk each day going to and from school?

error analysis: Item achievement was high. In 1977 it ranged from 73% to 82%. In four of the five items the most commonly chosen foil was obtained by using the wrong operation. All five items are "story problems".

Objective 2: Addition or subtraction — tenths or hundredths.

sample test item

Pearson Creek is usually 5.62 feet deep. After a heavy rain it was 5.96 feet deep. How much did the creek rise from the rain?

error analysis: In 1977, item achievement ranged from 71% to 81% on four items and 58% on one item. All of the items are "story problems".

Objective 3: Addition or subtraction — horizontal and vertical form.

sample test item

4.895 - .71 = ____

error analysis: Achievement was high when the items were written in vertical form. Item achievement when horizontal form was used ranged from 38%-60% with "ragged decimal" items lowest. The most frequent error involved decimal alignment when adding or subtracting.

DIAGNOSTIC TEST

1. .5
   +.3

2. Rewrite in vertical form.
   .6 + .3 = ____

3. .7 + .2 = ____

4. .5
   +.6

5. .8 + .4 = ____

6. .6
   -.2

7. Rewrite in vertical form.
   .7 - .2 = ____

8. .9 - .3 = ____

9. 1.2 - .4 = ____

10. 1.4 - .5 = ____

11. .23
+.54

12. .38
+.45

13. .67
+.54

14. .54 + .65 = ____

15. .63 + .37 = ____

16. 7.04 + .97 = ____

17. .67
-.41

18. .76 - .39 = ____

19. 10.04
- 7.18

20. Emmy Lou bought 7.9 gallons of gasoline on Monday and another 6.1 gallons on Friday. How much gasoline did she buy altogether?

21. If a casserole recipe calls for 2.5 pounds of hamburger and the smallest package at the meat counter weighs 2.89 pounds, how much extra hamburger would there be?

Estimate to determine which is the more reasonable answer.

22. 1.9 + 23.67 = ____
(a) 25 (b) 42

23. 2.3 + 37.46 = ____
(a) 60 (b) 40

24. 42.2 + .89 = ____
(a) 130 (b) 43

25. 36.2 - 1.36 = ____
(a) 23 (b) 35

Find the answer.

26. .4
+.16

27. .7
-.14

28. 25.6
- 3.47

29. 1.0
- .768

30. 1.9 + 21.68 = ____

31. 77.82 - 1.7 = ____

32. 11.8 - 7.46 = ____

33. 351.16 + .6 + 2.1 = ____

34. 3.256 + 7.1 + 5.0 = ____

35. 4
+7.02

36. 6
-1.13

37. 32.17 + 3.6 + 9 = ____

38. 18 - .69 = ____

TEST USE AND TEACHING SUGGESTIONS

Suggestion 1. Always inspect student work to determine the nature of the error.

Suggestion 2. Teach students to rewrite horizontal problems in vertical form, being careful to align decimals with precision in order to preserve the place value of the digits.

.7 + .5 = ____ .7
+.5

Items 1-10. (Add and subtract tenths, with and without regrouping over the decimal point.)

probable errors: Students will omit the decimal or place it incorrectly.

suggestions:

(a) Provide linear metric measurement activities to reinforce the meaning of tenths.

(b) Provide opportunities for reading and writing decimal numbers with tenths (hundredths, etc.)

(c) On items 2, 3, 5, 7, 8, and 10, provide extensive practice rewriting addition and subtraction problems in vertical form without finding solutions.

(d) Check the reasonableness of answers. Is:

.7 + .5 > 1?
.7 + .5 < 1?

(e) Provide work with metric and decimal place value charts.

METRIC

Decameter	Meter (1)	Decimeter ($\frac{1}{10}$)	Centimeter ($\frac{1}{100}$)	Millimeter ($\frac{1}{1000}$)
	(1.)	(.1)	(.01)	(.001)
	4.	2		
		2	3	
	1.	6	2	4

	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
(1)					.9		
(2)			2	0	.		
(3)					.0	4	
(4)					.0	3	7
(5)					.0	0	8
(6)				5	.0	6	

(a) Emphasize that the Ones Column is the pivotal point of this model.

(b) Read numbers and have students write them on the chart.

(1) nine tenths
(2) twenty
(3) four hundredths
(4) thirty-seven thousandths
(5) five and six hundredths
(6) eight thousands

Items 11-19. (Adding and subtracting hundredths with and without regrouping over the decimal point.)

probable errors:

(a) Students will omit the decimal or place it incorrectly.

(b) Students hesitate to regroup over the decimal point.

suggestion:

(a) Provide linear metric measurement activities to reinforce the meaning of tenths.

(b) Provide opportunities for reading and writing decimal numbers with tenths (hundredths, etc.)

(c) References to dollars and cents are useful here.

Items 20-21. (Using decimal numbers in story problems.)

suggestion: Focus on determining the operation and setting up the problem.

Items 22-25. (Estimating answers to addition and subtraction of decimal numbers.)

suggestion:

(a) In estimating the student should be aware that any number within a "reasonable" range is an acceptable answer.

(b) Provide opportunities for students to estimate orally with a partner, justifying answers; *or* ask them to write down *answers only*, doing all of the "work" mentally. *DO NOT* ALLOW them to compute first and then "round off" the answer!

Items 26-29. (Adding and subtracting ragged decimals, vertical form.)

probable errors: Students will probably either ignore any digits that are "ragged" or else they will try to add them to or subtract them from digits in other places.

suggestions: Have students append zeroes to the right of decimal digits as place holders.

Items 30-32. (Adding and subtracting "ragged" decimals, horizontal form.)

probable errors:

(a) See Items 26-29.
(b) Students will not align decimals.

suggestions:

(a) See Items 26-29.
(b) Use charts suggested for Items 1-10.

Items 33-34. (Adding "ragged" decimals including thousandths.)

suggestions: See items 26-32.

Items 35-36. (Adding and subtracting "ragged" decimals, including whole numbers without decimal points or zeroes, vertical form.)

suggestions: See items 26-29.

Items 37-38. (Adding and subtracting "ragged" decimals, including whole numbers, horizontal form.)

suggestions: See items 26-29 and 30-32.

DECIMALS

MULTIPLICATION — METRIC

RESULTS OF MEAP TESTING

There were two objectives tested — both at 10th grade.

Objective 1: Multiply a decimal fraction and a whole number power of ten.

sample test item
1000 x 6.23 = ____

error analysis: Item achievement ranged from 62%-69%. On each item approximately 20% of the students selected the response which had zeroes appended (6.23000).

Objective 2: Compute the product of two 2-place decimal fractions.

sample test item
23.48 x 71.56 = ____

error analysis: Item achievement ranged from 62%-72%. There was no discernible pattern in the errors made. As many students made multiplication errors as made mistakes in decimals placement.

remark: The approach to multiplication of decimals in this brochure assumes that the student has a working knowledge of metric measures and can find the area of rectangles. For metric measurement teaching suggestions refer to the Michigan Department of Education publication, *Teacher Resource Guide for Metric Education*.

DIAGNOSTIC TEST*

1. 1 dm = ____ m
2. 1 cm = ____ m
3. 1 m = ____ dm
4. 1 m = ____ cm
5. How many dm^2 in 1 m^2?
6. 1 m^2 = ____ dm^2
7. 1 dm^2 = ____ m^2
8. How many cm^2 in 1 m^2?
9. 1 m^3 = ____ cm^3
10. 1 cm^2 = ____ m^2

11. The length of Rectangle A is ____ cm = ____ dm.
12. The width of Rectangle A is ____ cm = ____ dm.
13. The area of Rectangle A is ____ cm^2 = ____ dm^2.

14. .4 x.3
15. .04 x .3
16. .04 x.03
17. .4 x 3

*This test was designed to help identify areas students might have difficulty with.

TEST USE AND TEACHING SUGGESTIONS

General Suggestion 1: Use one model consistently.

General Suggestion 2: Prior to work with multiplication of decimals it is necessary for the student to have a good knowledge of area measurement using metric measures.

Items 1-4. (Linear metric relationships.)

Be certain that the responses are in decimal form. Failure of any of these items demonstrates a lack of basic understanding of the metric system of measure. Instruction in metric measurement may be necessary prior to work with decimals.

Items 5-10. (Metric relationships of area measures.)

These relationships form the basis for multiplication of decimals. They must be learned if the techniques suggested here are to be used.

suggestion: A teaching sequence for remediation should include:

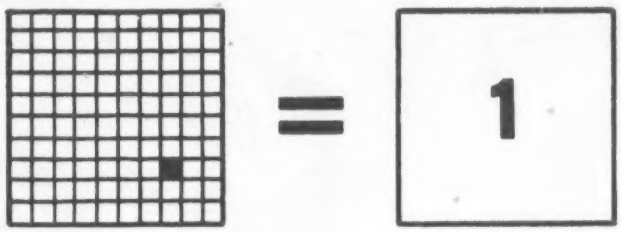
1. comparisons of the linear measures decimeter and centimeter with the meter.

2. making linear measures in dm and cm and expressing the results in centimeters, decimeters, and meters.

3. comparisons of square centimeter, square decimeter, and square meter. It is necessary for the student to make relationships such as: "If there are 100 square centimeters in 1 square decimeter then 1 square centimeter is .01 square decimeter (one of one hundred square centimeters in a square decimeter)."

   $100\ cm^2 = 1\ dm^2$

   Therefore $1\ cm^2 = .01\ dm^2$ . . . (one of one hundred units)



100 cm^2 1 dm^2

one of 100 (.01)

4. measuring rectangular objects to the nearest cm (dm) and finding the area in cm^2, dm^2 or m^2.

Items 11-13. (Area of a rectangle.)

The error at this level is most apt to occur with the area expressed in m^2.

suggestion 1. A teaching sequence for remediation should include:

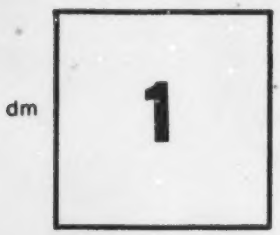
1. comparisons of the linear measures decimeter and centimeter with the meter.

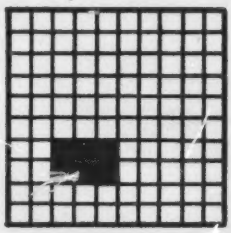
2. making linear measures in dm and cm and expressing them in centimeters, decimeters, and meters.

3. comparisons of square centimeter, square decimeter, and square meter.

4. Supply students with centimeter grids 1 dm x 1 dm. Have them find areas of rectangles by drawing the rectangles on the grid and counting the squares enclosed.

   Example:





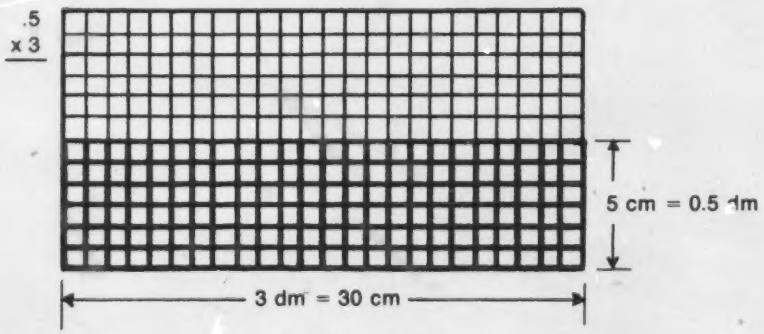
Rectangle A

(6 out of 100)
or .06

length: ____ cm = ____ dm
width: ____ cm = ____ dm
area: ____ cm² = ____ dm²

5. observing that the number of decimal positions in the product is the sum of the number of decimal positions in each of the factors.

suggestion 2. Multiplication of a whole number and a decimal follow immediately from rectangles with dimensions such as 5 cm by 3 dm (30 cm).



A = 150 cm² = 1.5 dm²
= 100 cm² + 50 cm²
= 1 dm² + .5 dm² (50 of 100)

Items 14-17. (Decimal multiplication.)

Failure to correctly place the decimal in any of these problems indicates a lack of ability to relate these problems to an area model. Time needs to be spent on tying these problems to the model suggested above.

DECIMALS

MULTIPLICATION

RESULTS OF MEAP TESTING

There were two objectives tested — both at 10th grade.

Objective 1: Multiply a decimal fraction and a whole number power of ten.

sample test item
1000 x 6.23 = ____

error analysis: Item achievement ranged from 62%-69%. On each item approximately 20% of the students selected the response which had zeroes appended (6.23000).

Objective 2: Compute the product of two 2-place decimal fractions.

sample test item
23.48 x 71.56 = ____

error analysis: Item achievement ranged from 62%-72%. There was no discernible pattern in the errors made. As many students made multiplication errors as made mistakes in decimal placement.

DIAGNOSTIC TEST

Complete:

1. $.25 = \frac{\quad}{100}$
2. $.1 = \frac{\quad}{10}$
3. $1.3 = \frac{\quad}{10}$
4. $1.3 = \frac{\quad}{100}$

Express as decimals:

5. $\frac{1}{100} =$ ____
6. $\frac{1}{1000} =$ ____
7. $\frac{250}{100} =$ ____

Complete:

8. $\frac{1}{10} \times 1 =$ ____
9. $\frac{1}{10} \times \frac{1}{10} =$ ____
10. $\frac{1}{100} \times \frac{1}{10} =$ ____
11. $.1 \times .1 = \frac{\quad}{10} \times \frac{\quad}{10}$
12. $.01 \times .1 = \frac{\quad}{100} \times \frac{\quad}{10}$
13. $.6 \times .7 = \frac{6}{\quad} \times \frac{\quad}{10}$
14. $1.1 \times .8 = \frac{\quad}{10} \times \frac{8}{\quad}$
15. When multiplying by .1 does the number get larger or smaller?
16. Is .14 x .8 larger or smaller than 10?
    larger or smaller than 1?
    larger or smaller than .1?
    larger or smaller than .01?

Supply decimal points in the answers:

17. .4 x .3 = 12

18.
$$\begin{array}{r} .7 \\ \times .9 \\ \hline 63 \end{array}$$

19.
$$\begin{array}{r} 4.4 \\ \times 6.1 \\ \hline 2684 \end{array}$$

20.
$$\begin{array}{r} 440 \\ \times\ 6.1 \\ \hline 2684 \end{array}$$

21.
$$\begin{array}{r} .44 \\ \times 6.1 \\ \hline 2684 \end{array}$$

22.
$$\begin{array}{r} .44 \\ \times .61 \\ \hline 2684 \end{array}$$

Complete:

23. .14 x .8 = ____

24. .23 x .61 = ____

25. 1.4 x .31 = ____

26.
$$\begin{array}{r} 1.04 \\ \times\ .31 \\ \hline \end{array}$$

27.
$$\begin{array}{r} .04 \\ \times .31 \\ \hline \end{array}$$

28.
$$\begin{array}{r} 2.3 \\ \times 5.1 \\ \hline \end{array}$$

TEST USE AND TEACHING SUGGESTIONS

General Suggestion. It is recommended that students be proficient in multiplication of fractions (unit fractions) prior to work with multiplication of decimals.

General Suggestion. It is recommended that students have a thorough understanding of the fractional and/or decimal form of the following: halves, thirds, fourths, fifths, tenths, hundredths and thousandths.

General Suggestion. If multiplication of decimal problems are written in horizontal form, have the student rewrite them in vertical form.

Items 1-7. (Rewriting decimals as fractions: Rewriting fractions as decimals.)

Missing any of these items indicates that the student needs more work with the conversion. This is learned by rote.

Items 8-10. (Products of powers of $\frac{1}{10}$.)

These problems form the basis of multiplication of decimals. Failure of any of them calls for remedial help with the multiplication of unit fractions.

suggestion: Use the fraction materials from this sequence.

Items 11-14. (Rewriting products of decimals as products of fractions.)

Understanding problems like these provides rationale for "moving the decimal point". This is an important transitional step. Additional work with rewriting decimals as fractions is necessary for students who have difficulty with these items.

Items 15-16. (Quanitification)

An error on either of these problems indicates that the student has no "feel" for the reasonableness of answers. This can be developed using the paper folding techniques illustrated for fractions and strategies for estimating.

Items 17-22. (Placement of the decimal.)

These six items test the students' ability to use the rule for placing the decimal. After justification, placement of the decimal should become mechanical.

Items 23-28. (Multiplication of decimals.)

If errors occur on problems 23-25, check to be certain that the student rewrote the problem in vertical form. Then determine whether the error is in multiplication or placement of the decimal. Errors on problems 26-28 are either errors in multiplication or in the placement of the decimal.

DECIMALS

DIVISION

For remediation it is suggested that a thorough development of "why" the decimal point is moved the same number of positions in the dividend and the divisor be presented only if requested by the student. Justification for solving an equivalent division problem with a whole number divisor should be made by using a hand calculator to solve each problem. This means giving assignments to (a) write problems with whole number divisors which are equivalent to those with decimal divisors and to (b) check that the quotients are the same with a calculator.

Example: $.06\overline{)72.8}$ $6\overline{)7280}$

RATIO & PROPORTION

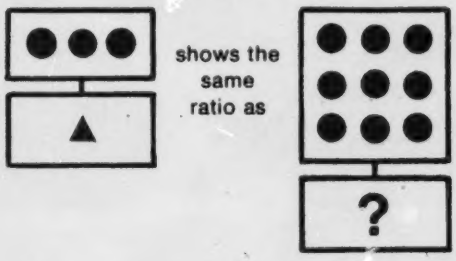
RESULTS OF MEAP TESTING

Three objectives have been tested on MEAP. One of the objectives is tested at 7th grade and two of the objectives are tested at 10th grade.

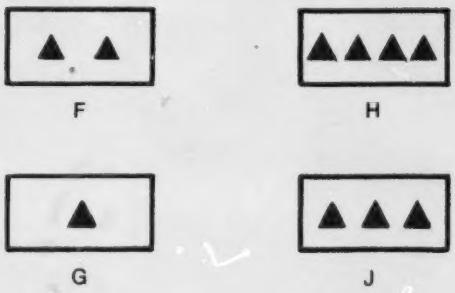
7TH GRADE MEAP

Objective: Identify pairs of sets depicting equivalent ratios.

sample test item



Which box completes the above ratio?



error analysis: Item achievement ranged from 45%-52%. The most frequently chosen foil was the one identical to that shown in the illustrated ratio.

10TH GRADE MEAP

Objective 1: Write a ratio describing the indicated comparison.

sample test item

What is the ratio of the shaded region to the *whole* region?

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\frac{3}{4}$

(d) $\frac{3}{5}$

error analysis: Item achievement was very high (90%-95%) on all of the items except the one shown. On that item 33% of the students selected the part-to-part ratio.

DIAGNOSTIC TEST — RATIO

Write each ratio as a fraction.

1. 3 to 7
2. 1 out of 4
3. 9 per 1

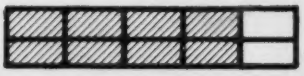
Use the table and write the ratio indicated.

A	A	B	C
A	A	B	C
A	A	B	C
A	B	B	C

4. the number of A's to C's.
5. the number of C's to B's
6. the number of A's to the total number of letters

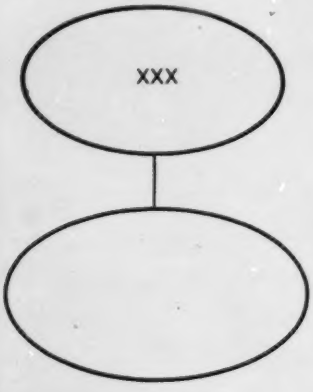
Write the ratio that compares the shaded part to the whole region.

7.

8. 

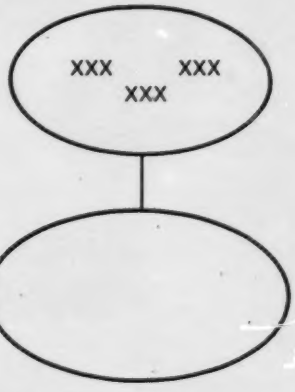
Complete the drawings, keeping the ratio of X's to 0's as $\frac{3}{4}$.

9



draw 0's here

10.



draw 0's here

Complete the tables. Keep the ratios the same.

10.

Frozen Orange Juice	1	2	3	4	5
Water	3	6			

11.

Cups of Flour	3	6	9	12	15	18
Cups of Sugar	2	4				

12. Write three ratios that are equal to $\frac{5}{2}$.

$\frac{5}{2} = ____ = ____ = ____$

13. Which ratio is equal to $\frac{9}{15}$?

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{81}{225}$

14. Which is a pair of equal ratios?

(a) $\frac{1}{3}, \frac{3}{5}$ (b) $\frac{4}{8}, \frac{2}{6}$ (c) $\frac{3}{12}, \frac{1}{9}$ (d) $\frac{6}{9}, \frac{4}{6}$

For each pair of ratios, test to see whether or not they are equal. Write *equal* or *not equal*.

15. $\frac{4}{6}, \frac{2}{3}$ _____

16. $\frac{1}{5}, \frac{40}{100}$ _____

17. $\frac{12}{27}, \frac{16}{36}$ _____

18. Write the ratio in *lowest terms* which is equal to $\frac{15}{20}$.

_____

19. There are 30 monkeys and 5 bears at the zoo. Write the ratio of monkeys to bears in *lowest terms*.

_____

PROPORTION

Find the missing number in each proportion.

20. $\frac{3}{6} = \frac{12}{\square}$

21. $\frac{25}{100} = \frac{\square}{4}$

22. $\frac{12}{18} = \frac{8}{\square}$

23. $\frac{5}{7} = \frac{9}{\square}$

Solve.

24-26. If 6 pounds of coffee cost $15.00, then

24. 12 pounds of coffee cost $____.
25. 3 pounds of coffee cost $____.
26. 1 pound of coffee costs $____.

27. There are three trucks for every seven cars in a parking lot. If there are 21 cars, how many trucks are there?

28. The library loses 5 books for every 2000 it lends. If the library loans 5200 books, how many books should it expect to lose?

TEST USE AND TEACHING SUGGESTIONS

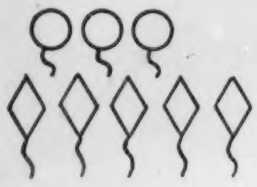
Items 1-8. (Write a ratio.)

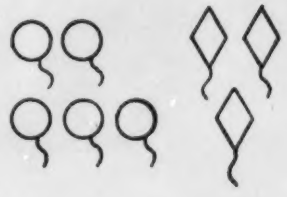
probable errors: Students will (a) reverse the order of the ratio or (b) compare a part to the whole when asked to compare part-to-part or vice versa.

suggestion: Have students choose the picture which corresponds to a given ratio.

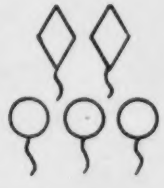
Example:

Which picture shows balloons to kites in the ratio 5 to 3?

(a) 

(b) 

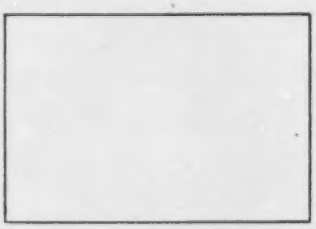
(c)

(d) 

suggestion: Have students draw a picture showing a given ratio.

Example:

Draw a picture of X's and 0's which shows the ratio of X's to 0's as $\frac{2}{5}$.



Draw Here

Items 9-12. (Indicate sets of equal ratios.)

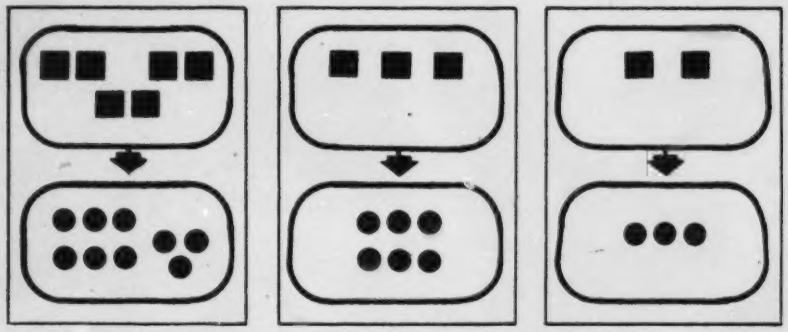
probable errors: Students will add the same amount to both terms of the ratio.

suggestion: Use real life situations which incorporate the concept of equal ratios.

suggestion: Have students discriminate unequal ratios.

Example:

Cross out the picture that *does not* show the same ratio of ■'s to ● 's as the others:



Cross out the ratio that *does not* belong in the table:

7	14	18	21
2	4	8	6

1	6	9	12
5	22	33	44

Items 13-17. (Determine whether or not two ratios are equal.)

probable errors: There are many possible errors. Check to see if the student knows at least one specific test for equivalence.

suggestion: Teach or review the cross-products ratio. That is, two ratios are equal if, and only if, their cross-products are equal.

Items 18-19. (Write a ratio in lowest terms.)

probable errors: The student will not reduce to lowest terms. The student may not know of a specific method to reduce a ratio to lowest terms.

suggestion: Review the general rule that the numerator and denominator of a fraction may be divided by the same number and the result is an equivalent fraction.

Example:

$$\frac{12}{16} \xrightarrow{\div 4} \frac{3}{4}$$

suggestion: Review *Greatest Common Factor.*

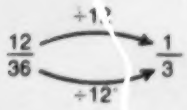
suggestion: Teach the method of successive differences. (Euclid's *Elements*, Book VII, Proposition 2.)

Example:

$$\frac{25}{37}$$

First difference: 37-25=12
Second difference: 25-12=13
Third difference: 13-12=1
Therefore 25 and 37 are relatively prime.

Example:



First difference: 36-12=24
Second difference: 24-12=12
Third difference: 12-12=0
Therefore 12 is a common factor.

Items 20-23. (Solve a proportion in equation form.)

probable errors: There are many common errors. They include:

(a) giving the factor as the missing term

Example:

$\frac{2}{3} = \frac{?}{15}$ Response: 5

(b) adding the factor

Example:

$\frac{2}{3} = \frac{?}{15}$ Response: 7

(c) adding the difference of the corresponding terms

Example:

$\frac{2}{3} = \frac{?}{15}$ Response: 14

15-3=12

suggestion: Teach the cross-products rule.

Items 24-28. (Solve a proportion given as a word problem.)

probable errors: Some students do not recognize the problem situation as arranging equal ratios. Many who do will not form the correct proportion to solve the problem.

suggestion: Have students form tables to set up proportions.

suggestion: Have students recognize the proportion that goes with a word problem.

Example:

Choose the proportion that fits the story:

There are 72 campers at camp. How many counselors are there if there are two counselors for every nine campers?

(a) $\frac{9}{2} = \frac{N}{72}$ (b) $\frac{N}{9} = \frac{72}{2}$ (c) $\frac{2}{9} = \frac{72}{N}$ (d) $\frac{N}{72} = \frac{2}{9}$

Note: $\frac{9}{2} = \frac{72}{N}$ is also correct.

PERCENT

RESULTS OF MEAP TESTING

Three objectives were tested — all at 10th grade.

Objective 1: Complete table of fractions, decimals, and percents.

sample test item
Choose the number which completes this table.

Fraction	Decimal	Percent
$\frac{1}{8}$		12.5%

(a) 0.25
(b) 0.18
(c) 1.25
(d) 0.125

error analysis: Item achievement ranged from 49% (item shown) to 81%. student errors indicate basic difficulties with both equivalences of percents and decimal , and equivalences of fractions and decimals.

Objective 2: Determine the percent of a given number.

sample test item
8% of 480 = _____

error analysis: On one item, 50% of 100 = _____, 70% of the students selected the correct response. On the other items 35% and 47% of the students selected the correct response. In each case nearly 30% of the students thought the correct response was obtained by dividing by the percent.

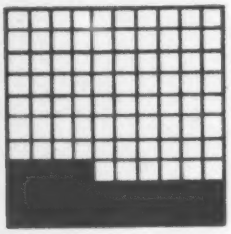
Objective 3: Verbal problem — find a given percent of a number.

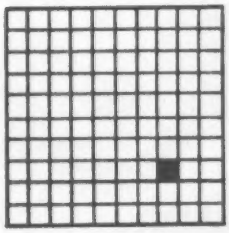
sample test item
What is 4% sales tax on $15.50?

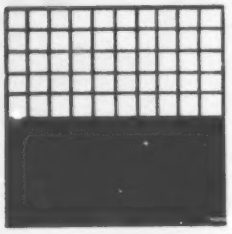
error analysis: Item achievement ranged from 47%-74%. Achievement was probably higher on these items because on one of the items division by the percent yielded the correct response and on another the quotient was not a possible response.

DIAGNOSTIC TEST

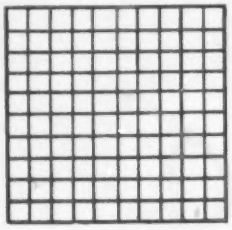
1. Write the percent represented by each figure:

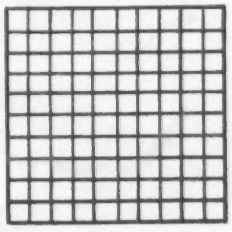
a.  ____%

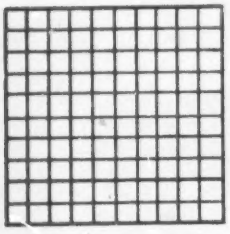
b.  ____%

c.  ____%

2. Shade areas on the figures to represent the given percent.

a.  ____7%

b.  ____72%

c.  ____30%

3. Write as percents:

$\frac{5}{100}$ = ____% $\frac{69}{100}$ = ____% $\frac{100}{100}$ = ____%

4. Write as percents:

.14 = ____% .98 = ____% .03 = ____%

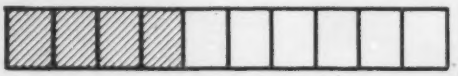
5. Write as a common fraction:

7% = ____ 23% = ____ 111% = ____

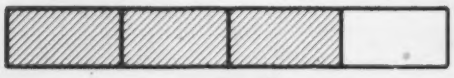
6. Write as a decimal:

19% = ____ 4% = ____ 125% = ____

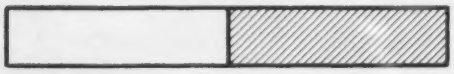
7. Write the percent represented by each figure.



____%



____%



____%

8. Shade approximately the given percent of each region.

60%

25%

100%

9. Which glass is about 80% full?

a.

b.

c.

d.

10. Write as a percent:

$\frac{2}{5}$ = ____% $\frac{9}{10}$ = ____% $\frac{4}{4}$ = ____%

11. Write as a percent:

.9 = ____% .125 = ____% 1.7 = ____%

12. Complete the table.

Fraction	Fraction with Denominator-100	Decimal	%
	$\frac{75}{100}$		
$\frac{1}{8}$			
		.84	
			$33\frac{1}{3}$ %

13. Write as a fraction in *lowest terms.*

a. 25% = ____ b. 30% = ____ c. 4% = ____

14. Circle the larger:

a. 20% or $\frac{1}{20}$ b. .4 or 4% c. 1% or 1

d. .75% or $\frac{4}{5}$

15. Circle the correct answer:

1% of 350 is (a) .35 (b) 3.5 (c) 35 (d) 350

100% of 600 is (a) .6 (b) 6 (c) 60 (d) 600

10% of 280 is (a) .028 (b) .28 (c) 2.8 (d) 28

16. 20% of 4000 is (a) 80000 (b) 200 (c) 80 (d) 800

50% of 300 is (a) 6 (b) 1500 (c) 150 (d) 15000

200% of 40 is (a) 5 (b) 8000 (c) 80 (d) 180

2% of 850 is (a) 17 (b) 170 (c) 425 (d) 1700

17. If Jack has 25% of the problems on this test wrong, what percent does he have correct?

18. Janeen made 80% of the free throws attempted. If she attempted 160 free throws, how many did she make?

TEST USE AND TEACHING SUGGESTIONS

General Suggestion: Emphasize percent as a ratio in which the second number is always 100.

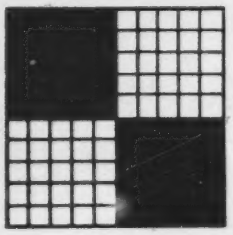
General Suggestion: Help students learn to express ratio as a decimal or a simple fraction.

Items 1-2. (Graphic representation of percent on the hundred square.)

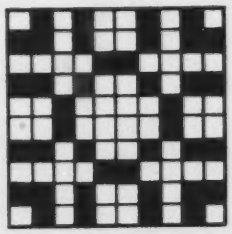
probable errors: Errors will result from a basic lack of understanding of percent.

suggestion: Provide opportunities to shade areas on the hundred square. Encourage students to be artistic.

Example:



50%



40%

Items 3-6. (Percent, hundredths fractions, hundredths decimals.)

probable errors: Errors result from lack of understanding that percent means per hundred or hundredths. Incorrect placement of the decimal is a common error.

suggestion: Read decimals, percents, and fractions aloud (properly) and have students record them as decimals, percents, and ratios.

Example:

$5\% \longrightarrow$ 5 percent

$.13 \longrightarrow$ 13 hundredths

$\frac{63}{100} \longrightarrow$ 63 hundredths

Items 7-9. (Graphic representation of percent.)

probable errors: Students will write the fraction rather than percent or will write the fraction followed by the percent symbol.

suggestion: Work with the hundred square.

Items 10-14. (Equivalent percents, decimals, and fractions.)

probable errors: Students will:

(a) change fractions to percents by writing the numerator and denominator in succession.

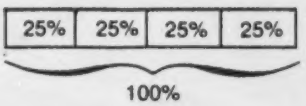
$\frac{2}{5} \longrightarrow 25\%$ or $\frac{2}{5} \longrightarrow .25$

(b) improperly place the decimal in conversion of percents to decimals and decimals to percents.

suggestion: Use paper strips folded as suggested in the fractions material and label in percents.

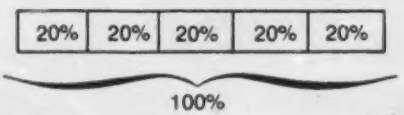
Example:

Fold fourths and label.



Example:

Fold fifths and label.



suggestion: Students should know percents for $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$.

suggestion: When converting a fraction to a percent use the 100 ratio.

$$\frac{2}{5} \overset{\times 20}{=} \frac{40}{100} = 40\%$$

Items 15-16. ("Percent of" problems.)

probable errors: Students will:

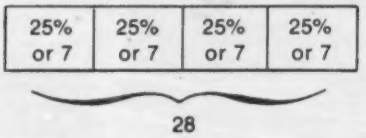
(a) improperly place the decimal.

(b) divided instead of multiplying.

suggestion: Use paper strip folding.

Example:

25% of 28 = ☐



suggestion: Use the proportion model.

Example:

25% of 28 $\qquad \frac{25\%}{100\%} = \frac{?}{28}$

Items 17-18. ("Percent of" word problems.)

probable errors: In problem 17 students may decide there is not enough data provided. In problem 18, as with all story problems, students will add, subtract, multiply, or divide. Those who do choose the correct operation are apt to improperly place the decimal.

suggestion: Teach the use of proportion in solving word problems.

suggestion: Assign problems in which the student is to "set up" the solution but not find it.

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